29 May -
2 June 2017

HIGGS BUNDLES AND RELATED TOPICS

Laboratoire J.A. Dieudonné

Parc Valrose - Nice

Titles and Abstracts

Tamas Hausel : Mirror symmetry with branes by equivariant Verlinde formulae

I will discuss an agreement of equivariant indices of semi-classical homomorphisms between pairwise mirror branes in the GL_2 Higgs moduli space on a Riemann surface. One side we have the components of the Lagrangian brane of U(1,1) Higgs bundles whose mirror was proposed by Nigel Hitchin to be certain even exterior powers of the hyperholomorphic Dirac bundle on the SL_2 Higgs moduli space. The agreement arises from a mysterious Hard Lefschetz type functional equation. This gives strong computational evidence for Hitchin's proposal. This is joint work with Anton Mellit and Du Pei.

Richard Wentworth : Deligne pairings and flat connections

Abstract: I will discuss a generalization of the construction of metrics and connections on Deligne pairings to the case of flat, not necessarily unitary, connections. In this context, there is an extension of Deligne's functorial Riemann-Roch isomorphism, and this gives an interpretation of the holomorphic extension of analytic torsion of Cappell-Miller. The hyperholomorphic line bundle on the twistor space of the moduli of Higgs bundles admits a meromorphic connection with certain properties. I will show that the existence of this connection for rank one Higgs bundles also follows from our construction. This is joint work with Gerard Freixas i Montplet.

Kang Zuo : Constructing crystalline representations on p-adic curves via stable Higgs bundles.

This is a joint work with R-R Sun and J-B Yang. Based on my previous work with Lan-Sheng we show that the category of rank-r periodic logarithmic Higgs bundles on a smooth logarithmic scheme (X, S)/Z_p after twisting line bundles is equivalent to the category of PGL_r-crystalline representations of the etale fundamental group of X\S/Q_p.

As applications we show:

1. Any hyperbolic curve carries irreducible PGL_2(F_p^f)- crystalline representations.

2. The projective line with n-marked points (n>3) carries infinetly many irreducible PGL_2(Z_p^f)-crystalline representations and such that the Zariski closure of the set of the corresponding periodic Higgs bundles has dimension n-3.

If time permits I will talk about some results on the finiteness of the set of isomorphic classes of GL_r(W_n(F_p^f))-crystalline representations on a given scheme with bounded r, n and f.

Pranav Pandit : Gradient flows, iterated logarithms, and semistability.

Conjecturally, the base of the Hitchin fibration can be interpreted as a space of stability structures on certain Fukaya-type categories. This conjectural picture is a special case of a more general local-to-global principle for stability structures on Fukaya categories with coefficients. I will tell the story of how, in the course of developing a formalism to address these conjectures, we came upon certain dynamical systems whose asymptotic behavior is controlled by iterated logarithms, and were led to the discovery of a novel filtration on semistable objects in a variety of contexts. This is based on joint work with Fabian Haiden, Ludmil Katzarkov, and Maxim Kontsevich.

Adrian Langer : Some remarks on opers.

I plan to survey a theory of opers and show a few new results concerning opers in a higher dimensional case.

Marina Logares : Donaldson-Uhlenbeck compactification of the moduli space of sheaves on a stack.

I will report on work in progress together with U. Bruzzo on a compactification of the moduli space of sheaves, and framed sheaves, on a Deligne-Mumford stack.

Indranil Biswas : Higgs bundles on Sasakian manifolds.

In this joint work with M. Mj we extend the Donaldson-Corlette-Hitchin-Simpson correspondence between Higgs bundles and flat connections on compact Kähler manifolds to compact quasi-regular Sasakian manifolds. A particular consequence is the translation of restrictions on Kähler groups proved using the Donaldson-Corlette-Hitchin-Simpson correspondence to fundamental groups of compact Sasakian manifolds.

Takuro Mochizuki : Mixed twistor D-modules and some examples.

In the study of mixed twistor D-modules, an important issue is to relate mixed twistor D-modules with concrete objects in various problems. Although we know that there exist many mixed twistor D-modules by an abstract existence theorem and by the functoriality, it is not easy to describe them explicitly, which is one of the blocks to find applications of the theory.

In this talk, after giving a brief survey of the general theory, we shall describe some examples of mixed twistor D-modules for which the underlying R-modules can be given explicitly. We shall also mention some examples for which V-filtrations can be computed, and we shall explain how they allow us to revisit some known results.

Motohico Mulase : An invitation to Quantum Curves and 2D TQFT.

Quantization of spectral curves appear in many examples of enumerative geometry problems and quantum topological invariants. The process of quantization is parallel (though not equivalent) to the passage from 2D TQFTs to CohFTs. In this talk, yet one more parallelism in Higgs bundles is explained. Here, spectral curves are Hitchin spectral curves, and quantum curves are opers. The talk is base on a recent joint paper with Olivia Dumitrescu.

Victoria Hoskins : Group actions on quiver moduli spaces.

We decompose the fixed loci of two types of actions on moduli spaces of quiver representations over a field k using group cohomology and give moduli-theoretic interpretations of the components. First, we consider actions by finite groups of quiver automorphisms and, over the complex numbers, we describe the symplectic and holomorphic geometry of these fixed loci in hyperkaehler quiver varieties in the language of branes. Second, for a perfect field k, we consider the action of the absolute Galois group on the points of

this quiver moduli space valued in an algebraic closure of k; the fixed locus is the set of k-rational points, and we obtain a decomposition of this fixed locus indexed by the Brauer group of k and we describe the rational points as quiver representations over division algebras. If time permits, I will explain how these techniques can also be employed to study moduli spaces of Higgs bundles. This is joint work with Florent Schaffhauser.

David Dumas : Geometric limits of convex RP^2 structures and cubic differentials.

We consider sequences going to infinity in the deformation space of convex RP^2 structures on a compact surface, and we study the relation between their developed images and the Labourie-Loftin parameterization of this deformation space by cubic holomorphic differentials on compact Riemann surfaces. By passing to a subsequence one can assume that the developed images converge to a (pointed) convex set in RP^2. When the associated Riemann surfaces remain in a compact subset of Teichmüller space, we show that any such pointed geometric limit in RP^2 is a convex polygon. We also show that the limit polygon is determined in an explicit way by the pattern and rate of collision of zeros of the sequence of cubic holomorphic differentials. This is joint work with Michael Wolf.

David Baraglia : On the image of the strongly parabolic Hitchin map.

A parabolic Higgs bundle is called strongly parabolic if the residue of the Higgs field at each puncture lies in the nilpotent radical of the corresponding parabolic. In joint work with M. Kamgarpour and R. Varma we have shown that the moduli space of strongly parabolic Higgs bundles is an integrable system. This motivates us to study the image of Hitchin map for strongly parabolic Higgs bundles. In this talk I will describe the image for arbitrary parabolics in the classical groups or G_2. Surprisingly, we find that the image is isomorphic to an affine space in all cases, except for certain "bad parabolics" in type D, where the image can be singular.

Laura Schaposnik : On some singular fibres of the Hitchin fibration.

The Hitchin fibration is a natural tool through which one can study the moduli space of Higgs bundles and its interesting subspaces (branes). We shall dedicate this talk to the study of certain singular fibres of the Hitchin fibrations, obtain correspondences between fibres, and provide a geometric description of branes which lie entirely over the singular loci (based partially on work in collaboration with David Baraglia, Steve Bradlow and Sebastian Heller).

Jorgen Andersen : The Verlinde formula for Higgs bundle moduli spaces.

In this talk we will present a Verlinde formula for the quantization of the Higgs bundle moduli spaces and stacks for any simple and simply-connected group. We further present a Verlinde formula for the quantization of parabolic Higgs bundle moduli spaces and stacks. We will explain how all these dimensions fit into a one parameter family of 2D TQFT's, encoded in a one parameter family of Frobenius algebras, which we will construct.

Vladimir Fock : Higher complex structures: spectral curve.

We suggest a geometric structure on smooth surfaces generalizing complex structures. Conjecturally the moduli space M of these structures coincide with higher Teichmüller spaces. In the talk we will discuss some natural properties of M and of its cotangent bundle. In particular we will construct an analogue of the Hitchin map from cotangent bundle to Lagrangian submanifolds of the cotangent bundle to the curve. This is joint work with A.Thomas.

Misha Verbitsky : Holomorphic bundles on hyperkahler manifolds and Tannakian categories.

Let M be a compact hyperkahler manifold with trivial Picard group (that is, generic in its deformation family), and C the tensor category of holomorpic vector bundles on M. Let C' be the category of holomorpic vector bundles on M', which is a deformation of M. Then C' is isomorphic to C as a tensor category. The Tannakian formalism associates a certain proalgebraic group to this tensor category. I would argue that this group plays the role of the fundamental group for hyperkahler geometry, and explain how one can use it to define analogue of Higgs bundles.

Qiongling Li : Higgs bundles of cyclic quiver type.

Given a G-Higgs bundle over a Riemann surface, there is a unique equivariant harmonic map from the universal cover of the Riemann surface into the associated symmetric space. We find a maximum principle theorem for a type of coupled linear elliptic systems and apply it to analyze the Hitchin equation for Higgs bundles of cyclic quiver type. In this case, the harmonic map is conformal and hence minimal. We show several domination results of the pullback metrics of the branched minimal immersion. This is joint work with Song Dai.

Ron Donagi : Hitchin's system and Geometric Langlands.

We discuss an approach to the Geometric Langlands Conjecture based on Hitchin's system combined with non Abelian Hodge theory. This is joint work with Pantev and Simpson.